

# CA14 ECONOMICS

## SECTION 3: CONSUMER THEORY

Cardinal utility, indifference curves, budget constraints, equilibrium, income and substitution effects, and consumer surplus

**Purpose.** This original DatalytlQs pack develops CON-01 to CON-05 from the KASNEB Revised Syllabus (June 2024) into examinable theory, calculations, diagrams and evidence tasks. It does not reproduce proprietary past-paper wording.

**Suggested study time:** 8-10 hours | **Assessment threshold:** 70% | **Prerequisite:** Demand and supply

### Learning outcomes

After this section, the learner should be able to apply diminishing marginal utility; calculate utility-maximising bundles using marginal utility per shilling; explain indifference-curve properties and marginal rate of substitution; derive and shift a budget line; identify tangency equilibrium; distinguish income and substitution effects; and calculate consumer surplus.

### Exam approach

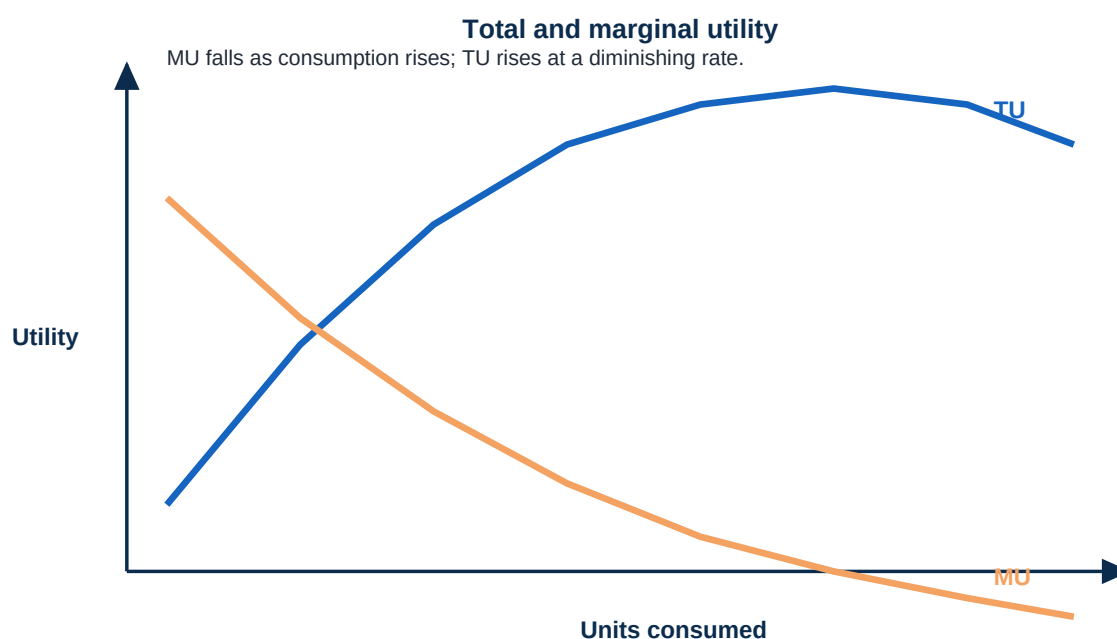
| Task                 | Required evidence                                                                        |
|----------------------|------------------------------------------------------------------------------------------|
| Utility calculation  | Correct marginal utility, MU/price comparison and reallocation logic.                    |
| Indifference diagram | Convex, downward-sloping curves; higher curve; no intersection; labelled bundle and MRS. |
| Budget problem       | Intercepts, slope, affordability and effect of income/price changes.                     |
| Equilibrium          | Tangency plus budget exhaustion; explain why a non-tangent bundle is not optimal.        |
| Evaluation           | State assumptions and distinguish analytical prediction from observed behaviour.         |

# 1. Consumer choice and utility

Consumer theory explains how a household allocates limited income among goods and services. Preferences describe rankings of alternatives; the budget constraint describes affordability. Equilibrium combines the most preferred attainable bundle with the financial constraint.

## 1.1 Cardinal utility

Cardinal analysis treats satisfaction as measurable in hypothetical utility units. **Total utility (TU)** is satisfaction from all units consumed. **Marginal utility (MU)** is the addition to total utility from one more unit:  $MU = \text{change in TU} / \text{change in quantity}$ .



| Units | Total utility | Marginal utility |
|-------|---------------|------------------|
| 0     | 0             | -                |
| 1     | 20            | 20               |
| 2     | 36            | 16               |
| 3     | 48            | 12               |
| 4     | 56            | 8                |
| 5     | 60            | 4                |
| 6     | 60            | 0                |
| 7     | 56            | -4               |

## Law of diminishing marginal utility

As successive units of a good are consumed within a period, marginal utility eventually declines, other factors held constant. TU rises while MU is positive, reaches a maximum when MU is zero and falls if MU becomes negative. The law concerns additional satisfaction, not necessarily total enjoyment.

**Exam distinction: Diminishing marginal utility is not the same as negative marginal utility. MU can decline from 20 to 16 to 12 while remaining positive; total utility still rises, but at a diminishing rate.**

## 2. Cardinal consumer equilibrium

With one good and money, a consumer buys units until the marginal utility of the last unit relative to price equals the marginal utility of money. With several goods, utility is maximised when the marginal utility per shilling is equalised across goods and the budget is exhausted:  $MU_x/P_x = MU_y/P_y = \dots = \text{marginal utility of money}$ .

### Worked example 1: marginal utility per shilling

| Unit | MU of X | $MU_x/P_x$ ( $P_x=20$ ) | MU of Y | $MU_y/P_y$ ( $P_y=10$ ) |
|------|---------|-------------------------|---------|-------------------------|
| 1    | 100     | 5.0                     | 60      | 6.0                     |
| 2    | 80      | 4.0                     | 50      | 5.0                     |
| 3    | 60      | 3.0                     | 40      | 4.0                     |
| 4    | 40      | 2.0                     | 30      | 3.0                     |
| 5    | 20      | 1.0                     | 20      | 2.0                     |

Suppose income is KES 80. Rank available units by MU per shilling: Y1=6, X1=5, Y2=5, Y3=4, X2=4, X3=3, Y4=3, and so forth. Select affordable units in descending order while respecting sequential consumption. One optimal bundle is X=2 units (KES 40) and Y=4 units (KES 40). The last purchased units each yield  $MU/P=3$ . The budget is exhausted and no feasible reallocation raises utility.

### Why equalisation works

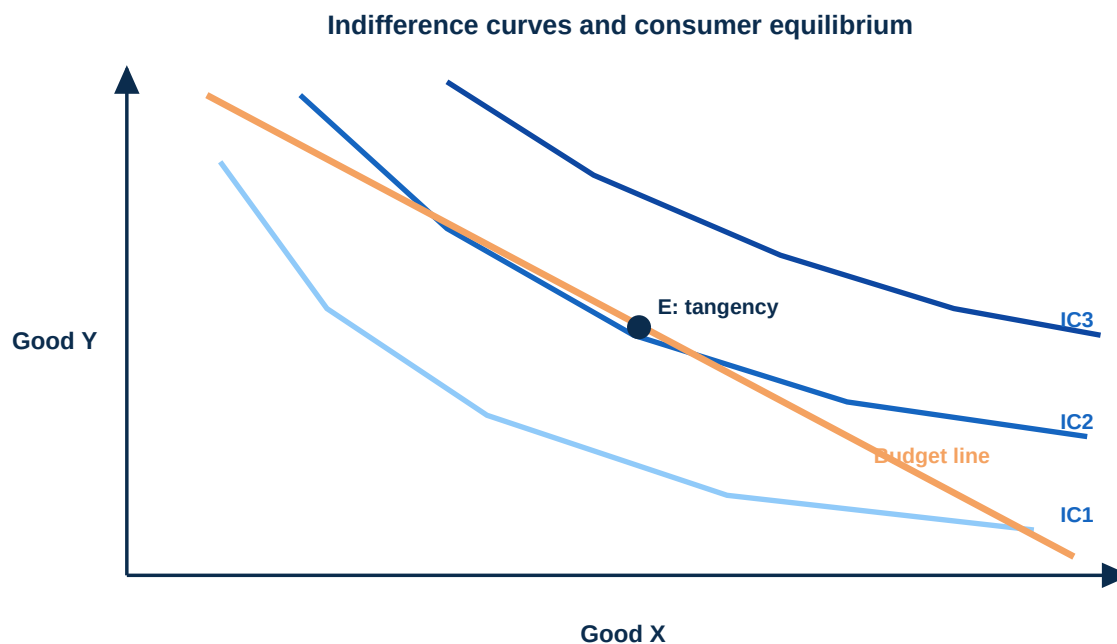
If  $MU_x/P_x$  exceeds  $MU_y/P_y$ , moving one shilling from Y toward X increases utility. Reallocation continues until the ratios are equal or indivisibility/budget constraints prevent exact equality.

### Assumptions and limitations

| Assumption                             | Limitation                                                    |
|----------------------------------------|---------------------------------------------------------------|
| Utility is cardinally measurable       | Satisfaction has no observable common unit.                   |
| Marginal utility of money is constant  | A purchase can change the value of remaining income.          |
| Consumers calculate consistently       | Information, cognition, habits and framing can alter choice.  |
| Goods are divisible and available      | Discrete purchases and stock-outs may prevent exact equality. |
| Preferences are stable during analysis | Tastes and social context may change.                         |

### 3. Ordinal utility and indifference curves

Ordinal analysis requires only that consumers can rank bundles. It does not assign numerical satisfaction. An indifference curve shows combinations of two goods that yield the same satisfaction, so the consumer is indifferent among points on the same curve.



#### Properties of well-behaved indifference curves

| Property                | Economic meaning                                                                       |
|-------------------------|----------------------------------------------------------------------------------------|
| Downward sloping        | With both goods desirable, more of one compensates for less of the other.              |
| Higher curves preferred | A bundle with more of at least one good and no less of the other is preferred.         |
| Do not intersect        | Intersection would imply inconsistent preference rankings under transitivity.          |
| Convex to the origin    | The consumer usually has diminishing willingness to substitute one good for the other. |
| Thin curves             | A bundle belongs to one satisfaction level, not a band of levels.                      |

#### Marginal rate of substitution

The **MRS of X for Y** is the amount of Y the consumer is willing to give up for one more unit of X while remaining equally satisfied. In absolute terms,  $MRS_{xy} = MU_x/MU_y$  and equals the slope magnitude of the indifference curve. Convexity reflects diminishing MRS: as X becomes abundant relative to Y, the consumer sacrifices progressively less Y for another X.

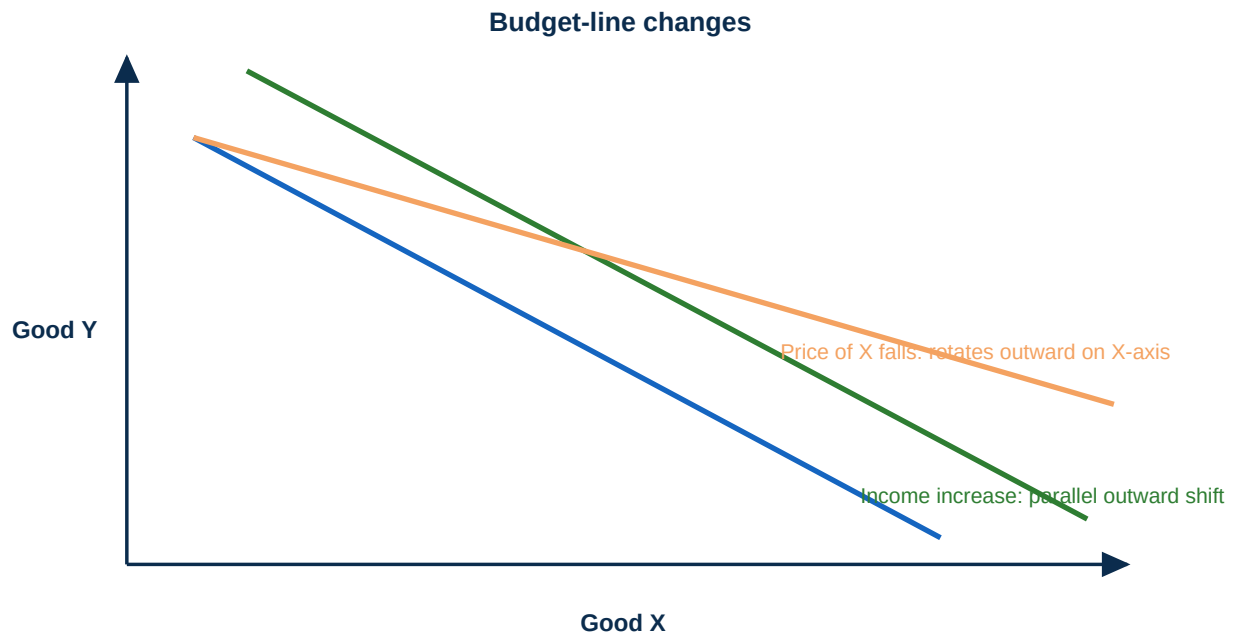
**Exam trap: A higher indifference curve represents greater satisfaction only if “more is preferred to less.” Do not attach cardinal utility numbers unless the question explicitly adopts a utility function.**

## 4. The budget constraint

For income  $M$ , prices  $P_x$  and  $P_y$ , the budget line is  $M = P_xX + P_yY$ . The X-intercept is  $M/P_x$ , the Y-intercept is  $M/P_y$  and the slope is  $-P_x/P_y$ . Points on the line exhaust income; points inside are affordable but leave income unspent; points outside are unaffordable.

### Worked example 2

Income  $M = \text{KES } 1,200$ ,  $P_x = \text{KES } 100$  and  $P_y = \text{KES } 200$ . The budget equation is  $1,200 = 100X + 200Y$ . X-intercept = 12 units; Y-intercept = 6 units; slope =  $-100/200 = -0.5$ . Bundle (4,4) costs  $400 + 800 = 1,200$  and lies on the line. Bundle (6,4) costs 1,400 and is unaffordable.



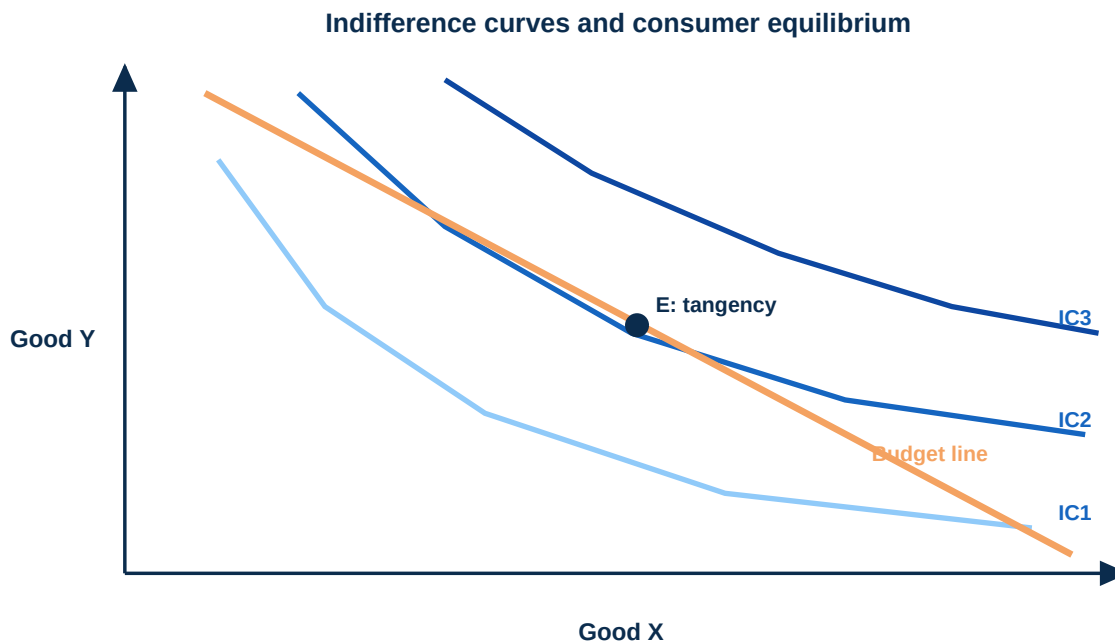
| Change                         | Income and effect                                               |
|--------------------------------|-----------------------------------------------------------------|
| Income rises; prices unchanged | Parallel outward shift; slope unchanged.                        |
| Income falls; prices unchanged | Parallel inward shift; slope unchanged.                         |
| Price of X falls               | X-intercept increases; line rotates outward around Y-intercept. |
| Price of Y rises               | Y-intercept decreases; line rotates inward around X-intercept.  |
| Both prices and income double  | Real opportunity set unchanged; same line.                      |

### Opportunity cost in the budget slope

The absolute slope  $P_x/P_y$  is the market opportunity cost of X in units of Y. In the example, one additional X costs half a unit of Y. This market trade-off is compared with the consumer's subjective trade-off, MRS, at equilibrium.

## 5. Ordinal consumer equilibrium

The consumer selects the highest attainable indifference curve. For a smooth interior solution, equilibrium occurs where the budget line is tangent to an indifference curve:  $MRS_{xy} = P_x/P_y$ , and the budget is exhausted. Equivalently,  $MU_x/P_x = MU_y/P_y$ .



### Why tangency matters

If  $MRS_{xy}$  exceeds  $P_x/P_y$ , the consumer values X relative to Y more highly than the market does and can gain by buying more X and less Y. If  $MRS_{xy}$  is below  $P_x/P_y$ , the consumer can gain by buying more Y. At equality, no marginal reallocation raises satisfaction.

### Corner solutions

Tangency is not universal. If preferences or relative prices make one good consistently more attractive, the optimum may occur at an intercept, with only one good purchased. A complete answer therefore states that tangency describes an interior solution under differentiable, convex preferences.

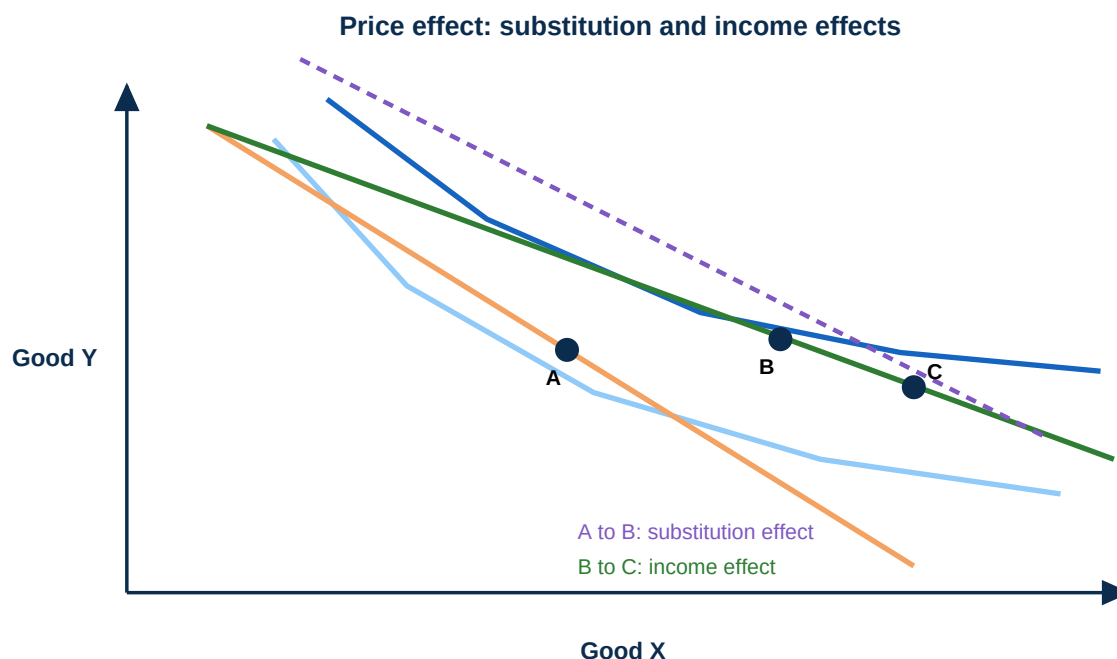
### Worked example 3: Cobb-Douglas utility

Let utility  $U = X^{0.5}Y^{0.5}$ , income  $M=400$ ,  $P_x=20$  and  $P_y=10$ . For equal exponents, the consumer spends half of income on each good. Expenditure on X=200, so  $X=10$ ; expenditure on Y=200, so  $Y=20$ . Check: total spending =  $20(10)+10(20)=400$ . At this bundle,  $MRS = Y/X = 2$  and  $P_x/P_y = 2$ .

**Full-credit equilibrium answer: State both conditions: the marginal trade-off equals the price ratio, and the chosen bundle is affordable with the budget exhausted.**

## 6. Price, substitution and income effects

When the price of X falls, X becomes cheaper relative to Y and the consumer's real purchasing power rises. The total change in X demanded - the price effect - can be separated into a substitution effect and an income effect.



| Effect              | Definition                                                                                                  | Direction for a price fall in X                                     |
|---------------------|-------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| Substitution effect | Change caused by the new relative price while holding utility or purchasing power appropriately compensated | Toward X; X is relatively cheaper.                                  |
| Income effect       | Change caused by increased real purchasing power after the compensated response                             | Toward X if X is normal; away from X if X is inferior.              |
| Price effect        | Combined observed movement from initial to final optimum                                                    | Normally increases X; may fall only in the exceptional Giffen case. |

### Normal, inferior and Giffen cases

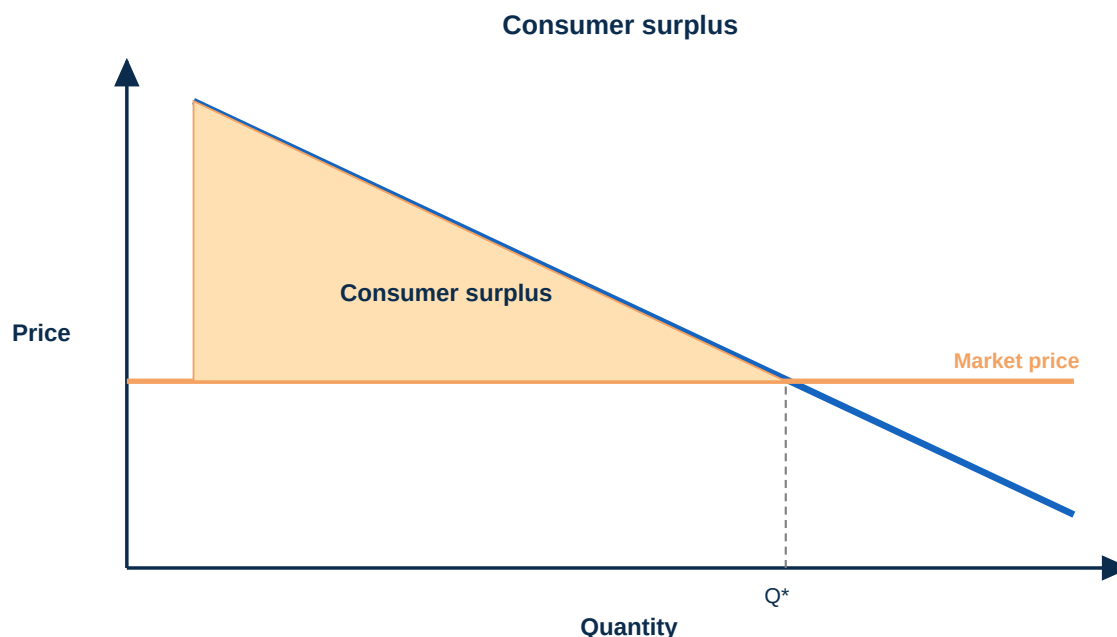
For a normal good, substitution and income effects reinforce each other after a price fall. For an inferior good, the income effect opposes the substitution effect, but the substitution effect is normally stronger, so demand still slopes downward. A Giffen good is an inferior good for which the opposing income effect exceeds the substitution effect, producing an upward-sloping demand response over the relevant range. Inferior does not automatically mean Giffen.

### Hicks and Slutsky compensation

Hicksian analysis adjusts income to keep the consumer on the original indifference curve. Slutsky compensation adjusts income so the original bundle remains just affordable. They use different compensation rules but both isolate the substitution response from the real-income response.

## 7. Consumer surplus

Consumer surplus is the difference between the maximum amount consumers are willing to pay and the amount actually paid. It measures net private benefit from exchange under standard assumptions.



### Calculation

For a linear demand curve with choke price KES 500, market price KES 200 and quantity 60 units, consumer surplus is the triangular area above price and below demand:  $0.5 \times (500 - 200) \times 60 = \text{KES } 9,000$ . State the currency and explain that this is an approximate monetary measure of benefit.

### Applications and limitations

| Use                                                                                                                                                                                                                                                                                                         | Limitations                                                                                  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| Welfare effect of a price change                                                                                                                                                                                                                                                                            | Change in surplus is informative but may not capture income distribution.                    |
| Project or policy appraisal                                                                                                                                                                                                                                                                                 | Benefit estimates depend on demand quality and counterfactual assumptions.                   |
| Tax analysis                                                                                                                                                                                                                                                                                                | Loss of surplus combines transfers and deadweight loss; not every loss is a net social loss. |
| Market comparison                                                                                                                                                                                                                                                                                           | Interpersonal utility comparisons and income effects complicate interpretation.              |
| <b>Area discipline: Identify the geometric shape before applying a formula. A linear triangular area uses <math>0.5 \times \text{base} \times \text{height}</math>; a rectangle uses <math>\text{base} \times \text{height}</math>. Do not add currency values to quantities without interpreting axes.</b> |                                                                                              |

## 8. Integrated worked case

**Case.** A learner has KES 600 for data bundles (X) priced at KES 100 and textbooks (Y) priced at KES 200. Preferences are convex and both goods are desirable.

| Requirement            | Model analysis                                                             |
|------------------------|----------------------------------------------------------------------------|
| Budget equation        | $600 = 100X + 200Y$ .                                                      |
| Intercepts and slope   | X-intercept 6; Y-intercept 3; slope -0.5.                                  |
| Affordable bundle test | (2,2) costs 600 and lies on the line; (3,2) costs 700 and is outside.      |
| Equilibrium condition  | At an interior optimum $MRS_{xy} = P_x/P_y = 0.5$ and spending equals 600. |

| Requirements           | Model analysis                                                                     |
|------------------------|------------------------------------------------------------------------------------|
| Income rise to 800     | Parallel outward shift; intercepts become 8 and 4.                                 |
| Price of X falls to 75 | Line rotates outward on X-axis; X-intercept becomes 8 while Y-intercept remains 3. |

## 9. Original assessment items

### Question 1 - Utility

Total utility from the fourth unit is 70 and from the third is 60. Calculate marginal utility of the fourth unit.

**Answer:**  $MU_4 = 70 - 60 = 10$  utility units.

### Question 2 - Allocation

$MU_x/P_x = 6$  while  $MU_y/P_y = 4$ . What reallocation raises utility?

**Answer:** Shift expenditure from Y toward X, subject to divisibility and the budget, until the ratios equalise.

### Question 3 - Budget line

$M=900$ ,  $P_x=150$ ,  $P_y=100$ . Derive the equation, intercepts and slope.

**Model answer:**  $900=150X+100Y$ ; X-intercept=6; Y-intercept=9; slope=-1.5. One X costs 1.5 units of Y.

### Question 4 - Indifference curves

Explain why two indifference curves cannot intersect.

**Model answer:** Intersection would make the common bundle indifferent to bundles on both curves. Transitivity would then imply those bundles are indifferent to each other, contradicting the claim that one lies on a higher satisfaction curve.

### Question 5 - Evaluation

Distinguish an inferior good from a Giffen good.

**Model answer:** An inferior good has a negative income effect: demand falls as real income rises. A Giffen good is a special inferior good whose opposing income effect exceeds the substitution effect, so quantity demanded rises when its price rises. Most inferior goods are not Giffen.

### Question 6 - Surplus

Choke price is 120, market price 60 and quantity 40. Calculate consumer surplus.

**Answer:**  $0.5 \times (120-60) \times 40 = 1,200$  monetary units.

## 10. Evidence and mastery requirements

| Evidence ID | Required output                     | Acceptance criterion                                                    |
|-------------|-------------------------------------|-------------------------------------------------------------------------|
| E3.1        | Utility schedule and graph          | TU/MU correct; satiation and negative MU identified.                    |
| E3.2        | Two-good MU-per-shilling allocation | Sequential units ranked; budget exhausted; equilibrium explained.       |
| E3.3        | Indifference map and budget line    | Curve properties, intercepts, slope and tangency accurately labelled.   |
| E3.4        | Income/substitution decomposition   | Initial, compensated and final equilibria plus effect directions shown. |
| E3.5        | Consumer-surplus calculation        | Shape, formula, units and welfare interpretation correct.               |

### Mastery checklist

| Item                                                    | Mastered |
|---------------------------------------------------------|----------|
| calculate marginal utility and identify diminishing MU. |          |
| allocate income using MU per shilling.                  |          |

| Learn                                                                                                                                                                | Teacher |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|
| explain indifference-curve properties and MRS.                                                                                                                       |         |
| derive a budget line and analyse income or price changes.                                                                                                            |         |
| state both conditions for consumer equilibrium.                                                                                                                      |         |
| separate substitution and income effects.                                                                                                                            |         |
| calculate and interpret consumer surplus.                                                                                                                            |         |
| <b>Progression rule: Complete E3.1-E3.5 before the section quiz. A score below 70% triggers remediation against failed CON reference IDs before another attempt.</b> |         |

## References and provenance

KASNEB. *Certified Public Accountants (CPA) Revised Syllabus, June 2024*, Paper No. 4: Economics. This pack is original DatalytlQs instructional content prepared for syllabus traceability and learner assessment; it is not an official KASNEB publication.

Source links: <https://www.kasneb.or.ke/cpa/> | <https://sgb.ac.ke/wp-content/uploads/2024/12/CPA-Syllabus-Reviewed-June-2024.pdf>